ELECTROMAGNETIC WAVES

EE 3321 Electromagnetic Field Theory

Pioneering 21st Century Electromagnetics and Photonics

http://emlab.utep.edu

Maxwell's Equations

$$\nabla \bullet \vec{D} = 0 \qquad \vec{D}, \vec{E} \perp \vec{k}$$

$$\nabla \bullet \vec{B} = 0 \qquad \vec{B}, \vec{H} \perp \vec{k}$$

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E}$$

Predicts waves

Wave Equation

Helmholtz Wave Equation

$$\nabla^2 u + \left(\frac{\omega}{v}\right)^2 u = 0$$

$u \equiv$ disturbance
$\omega \equiv$ frequency
$v \equiv$ velocity

Inhomogeneous Media

$$\nabla \times \left(\frac{1}{\mu}\nabla \times \vec{E}\right) = \omega^2 \varepsilon \vec{E}$$

Used mostly in numerical analysis.

Homogeneous Media

$$\nabla^2 \vec{E} + \omega^2 \mu\varepsilon \vec{E} = 0$$

Used mostly in closed-form analysis.

$$k^2 = \left(\frac{\omega}{v}\right)^2 = \omega^2 \mu\varepsilon$$

$k \equiv$ wave number

EM Wave Velocity

$$v = 1/\sqrt{\mu\varepsilon} \qquad c_0 = 1/\sqrt{\mu_0\varepsilon_0}$$

$$v = c_0/n$$

$$n = \sqrt{\mu_r \varepsilon_r}$$

$c_0 = 299{,}792{,}458$ m/s
$n \equiv$ refractive index

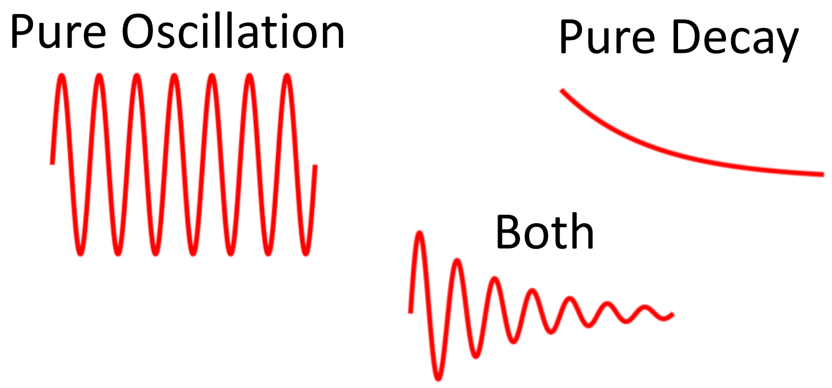
Solution to Wave Equation

$$\nabla^2 \vec{E} + k^2 \vec{E} = 0 \rightarrow \begin{matrix} \nabla^2 E_x + k^2 E_x = 0 \\ \nabla^2 E_y + k^2 E_y = 0 \\ \nabla^2 E_z + k^2 E_z = 0 \end{matrix} \rightarrow E_i(z) = Ae^{-jkz} + Be^{+jkz}$$

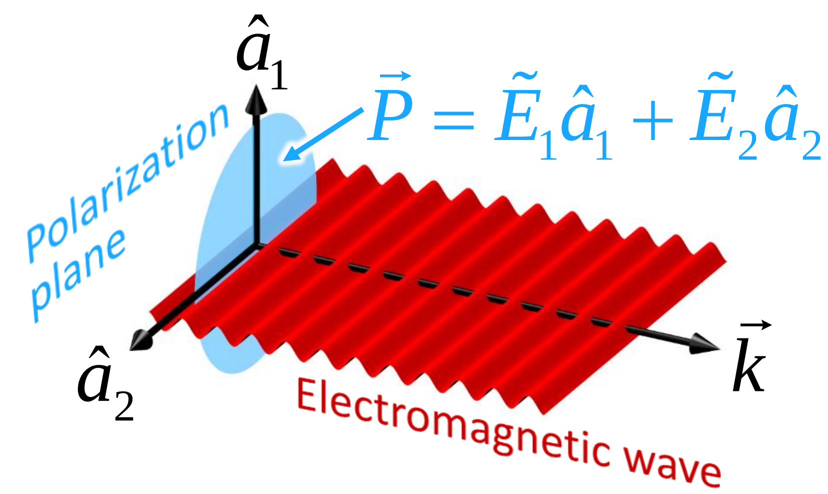
Solution is complex exponentials.

Forward Wave — Backward Wave

Waves can only do 2.5 things:



Plane Waves



Time-Domain

$$\vec{E}(t) = \vec{P}\cos\left(\omega t - \vec{k} \bullet \vec{r}\right)$$

Frequency-Domain

$$\vec{E}(\omega) = \vec{P}\exp\left(-j\vec{k} \bullet \vec{r}\right)$$

Wave Vector

$|\vec{k}| = 2\pi/\lambda$ — Conveys wavelength λ inside medium

$|\vec{k}| = k_0 = 2\pi/\lambda_0$ — In vacuum.

$|\vec{k}| = k_0 n$ — Also conveys refractive index n when frequency is known.

Relation Between E&H

Directionality: $\vec{E} \perp \vec{k} \perp \vec{H}$

Magnetic Field

$$\vec{H}(\omega) = \frac{\vec{k} \times \vec{P}}{\omega\mu}\exp\left(-j\vec{k} \bullet \vec{r}\right)$$

Impedance

$$\eta = \frac{E_0}{H_0} = \sqrt{\frac{\mu/\varepsilon}{1 + \sigma/j\omega\varepsilon}}$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + (\sigma/\omega\varepsilon)^2\right]^{1/4}}$$

$$\measuredangle \eta = 0.5\tan(\sigma/\omega\varepsilon)$$

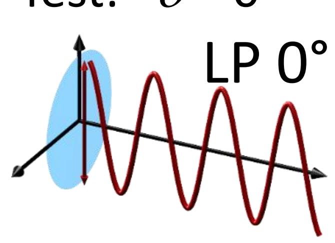
Polarization

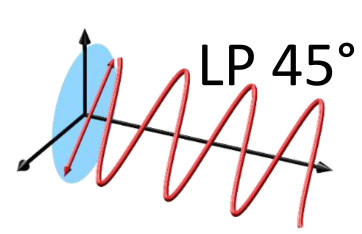
$$\vec{P} = \left(E_1\hat{a}_1 + E_2 e^{j\delta}\hat{a}_2\right)e^{j\theta}$$

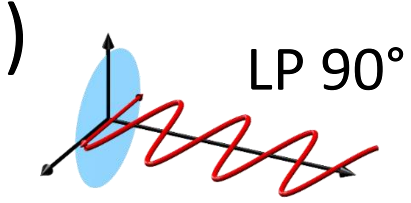
Expanded Polarization Vector

Linear Polarization (LP)

Test: $\delta = 0°$



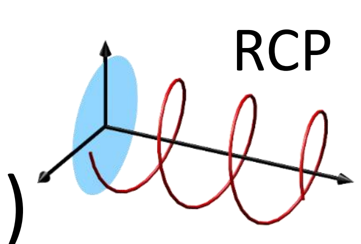


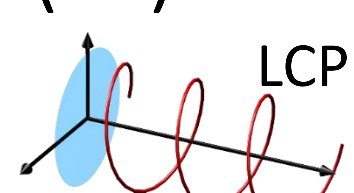


Circular Polarization (CP)

Test: $\delta = +90°$ and $E_1 = E_2$

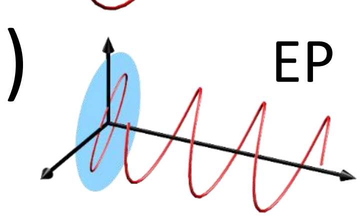
Right-Hand CP (RCP): $\delta = +90°$
Left-Hand CP (LCP): $\delta = -90°$





Elliptical Polarization (EP)

Test: not LP or CP
LP and CP are just special cases of EP.



Properties

Loss Tangent

$$\tan\delta = \varepsilon''/\varepsilon' \qquad P(z) = P_0 e^{-k\delta z}$$

Propagation Constant

$$\gamma = \alpha + j\beta \qquad E(z) = E_0 e^{-\gamma z}$$

Attenuation Coefficient

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + (\sigma/\omega\varepsilon)^2} - 1\right]}$$

Phase Constant

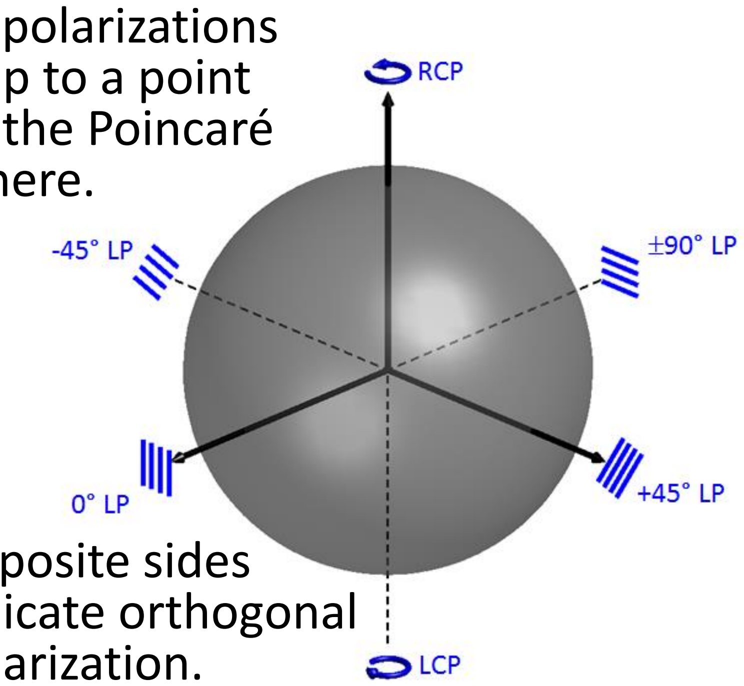
$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + (\sigma/\omega\varepsilon)^2} + 1\right]}$$

Absorption Coefficient

$$\alpha_{abs} = k\delta = 2\alpha \qquad P(z) = P_0 e^{-\alpha_{abs} z}$$

Poincaré Sphere

All polarizations map to a point on the Poincaré sphere.



Opposite sides indicate orthogonal polarization.