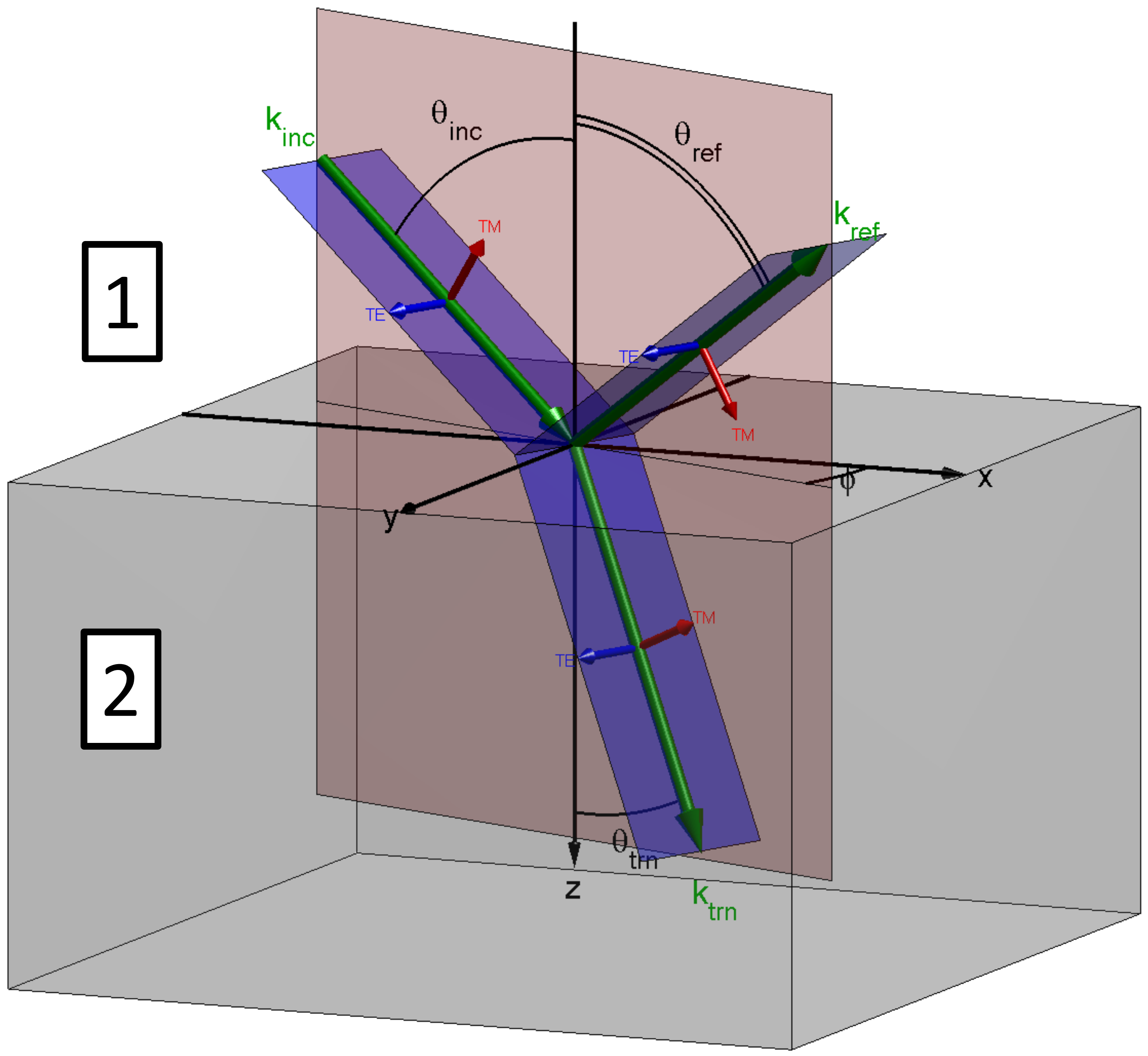
SCATTERING AT AN INTERFACE

EE 4347 Applied Electromagnetics

Pioneering 21st Century Electromagnetics and Photonics

http://emlab.utep.edu/

Geometry and Polarization



$$\hat{a}_{\mathrm{TE}} = \frac{\hat{a}_z \times \vec{k}_{inc}}{\left|\hat{a}_z \times \vec{k}_{inc}\right|} \qquad \hat{a}_{\mathrm{TM}} = \frac{\hat{a}_{\mathrm{TE}} \times \vec{k}_{inc}}{\left|\hat{a}_{\mathrm{TE}} \times \vec{k}_{inc}\right|}$$

Law of Reflection and Refraction

$$\theta_{\mathrm{ref}} = \theta_{\mathrm{inc}} \qquad n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{(Snell's Law)}$$

Normal Incidence (θ_1 = 0°)

$$r = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad t = \frac{2\eta_2}{\eta_2 + \eta_1} \qquad 1 + r = t$$

$$R = |r|^2 \qquad T = |t|^2 \frac{\eta_1}{\eta_2} \qquad R + T = 1$$

There is no distinction between TE and TM for LHI media.

Fresnel Equations (Amplitude)

$$r_{\mathrm{TE}} = \frac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2} \qquad t_{\mathrm{TE}} = \frac{2\eta_2\cos\theta_1}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2} \qquad 1 + r_{\mathrm{TE}} = t_{\mathrm{TE}}$$

$$r_{\mathrm{TM}} = \frac{\eta_2\cos\theta_2 - \eta_1\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1} \qquad t_{\mathrm{TM}} = \frac{2\eta_2\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1} \qquad 1 + r_{\mathrm{TM}} = t_{\mathrm{TM}}\frac{\cos\theta_2}{\cos\theta_1}$$

Reflectance and Transmittance (Power)

$$R_{\mathrm{TE}} = |r_{\mathrm{TE}}|^2 \qquad T_{\mathrm{TE}} = |t_{\mathrm{TE}}|^2 \frac{\eta_1\cos\theta_2}{\eta_2\cos\theta_1} \qquad R_{\mathrm{TE}} + T_{\mathrm{TE}} = 1$$

$$R_{\mathrm{TM}} = |r_{\mathrm{TM}}|^2 \qquad T_{\mathrm{TM}} = |t_{\mathrm{TM}}|^2 \frac{\eta_1\cos\theta_2}{\eta_2\cos\theta_1} \qquad R_{\mathrm{TM}} + T_{\mathrm{TM}} = 1$$

$$\frac{T_{\mathrm{TE}}}{|t_{\mathrm{TE}}|^2} = \frac{T_{\mathrm{TM}}}{|t_{\mathrm{TM}}|^2}$$

Brewster's Angle

$$\sin^2\theta_B\Big|_{\mathrm{TE}} = \frac{1 - \mu_{r1}\varepsilon_{r2}/\mu_{r2}\varepsilon_{r1}}{1 - \left(\mu_{r1}/\mu_{r2}\right)^2}$$

No Brewster's angle for TE polarization if there is no magnetic response.

$$\sin^2\theta_B\Big|_{\mathrm{TM}} = \frac{1 - \mu_{r2}\varepsilon_{r1}/\mu_{r1}\varepsilon_{r2}}{1 - \left(\varepsilon_{r1}/\varepsilon_{r2}\right)^2}$$

$$\tan\theta_B\Big|_{\mathrm{TM}} = \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}} = \frac{n_2}{n_1}$$

for no magnetic response

Critical Angle

$$\theta_c = \sin^{-1}\left(n_2/n_1\right)$$