4-8

设 $K=\begin{pmatrix}k_{11} & k_{12} & \cdots & k_{15}\\ k_{21} & \cdots & & k_{25}\\ k_{31} & \cdots & & k_{35}\end{pmatrix}$ $\quad A+BK=\begin{pmatrix}0 & 1 & 0 & 2 & 1\\ 0 & 0 & 1 & 4 & 3\\ 7+k_{31} & 6+k_{32} & 4+k_{33} & 3+k_{34} & 1+k_{35}\\ 8+2k_{21} & 7+2k_{22} & 5+2k_{23} & 9+2k_{24} & 3+2k_{25}\\ 5-k_{11} & 8-k_{12} & 6-k_{13} & 2-k_{14} & 9-k_{15}\end{pmatrix}$

令 $8+2k_{21}=7+2k_{22}=5+2k_{23}=0$

$5-k_{11}=8-k_{12}=6-k_{13}=0$ $\quad$ 则 $A+BK=\left(\begin{array}{ccc|cc}0 & 1 & 0 & 2 & 1\\ 0 & 0 & 1 & 4 & 3\\ 7+k_{31} & 6+k_{32} & 4+k_{33} & 0 & 0\\ \hline 0 & 0 & 0 & -2 & 0\\ 0 & 0 & 0 & 0 & -3\end{array}\right)=\begin{pmatrix}A_{11} & A_{12}\\ 0 & A_{22}\end{pmatrix}$

~~$9+2k_{24}=0$~~, ~~$9-k_{15}=0$~~

$3+k_{34}=1+k_{35}=0$, $3+2k_{25}=2-k_{14}=0$

$9+2k_{24}=-2$, $9-k_{15}=-3$.

对左上角块配置即可. $|\lambda I-A_{11}|=\lambda^3-(k_{33}+4)\lambda^2-(k_{32}+6)\lambda-(k_{31}+7)$

期望多项式为 $(\lambda+4)(\lambda+1+j)(\lambda+1-j)=\lambda^3+6\lambda^2+16\lambda+8$

$\therefore k_{33}=-10$, $k_{32}=-16$, $k_{31}=-15$

综上. $K=\begin{pmatrix}5 & 8 & 6 & 2 & 12\\ -4 & -3.5 & -2.5 & -5.5 & -1.5\\ -15 & -16 & -10 & -3 & -1\end{pmatrix}$

4-9 斜坡输入. 因此需要两个积分器.

$$\begin{cases}\dot q_1=e=y-y_r\\ \dot q_2=q_1\end{cases}$$

系统方程为:
(A·)

$$\begin{pmatrix}\dot x\\ \dot q_1\\ \dot q_2\end{pmatrix}=\begin{pmatrix}A & 0 & 0\\ C & 0 & 0\\ 0 & I & 0\end{pmatrix}\begin{pmatrix}x\\ q_1\\ q_2\end{pmatrix}+\begin{pmatrix}B\\ 0\\ 0\end{pmatrix}u+\begin{pmatrix}d\\ -y_r\\ 0\end{pmatrix}\qquad y=Cx=[C\ 0\ 0]\begin{pmatrix}x\\ q_1\\ q_2\end{pmatrix}$$

状态反馈为: $u=[k_1\ k_2\ k_3]\begin{pmatrix}x\\ q_1\\ q_2\end{pmatrix}=k_1x+k_2q_1+k_3q_2$

闭环系统方程为：
$$\begin{pmatrix} \dot{x} \\ \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} = \begin{pmatrix} A+BK_1 & BK_2 & BK_3 \\ C & 0 & 0 \\ 0 & I & 0 \end{pmatrix} \begin{pmatrix} x \\ q_1 \\ q_2 \end{pmatrix} + \begin{pmatrix} d \\ -y_r \\ 0 \end{pmatrix}$$

(B.)

$$y = [C \ 0 \ 0] \begin{pmatrix} x \\ q_1 \\ q_2 \end{pmatrix}$$

定理 4-7. (A.) 可控的充要条件是 4-18 可控. ~~且 rank~~ $\cancel{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = n+}$

且 $\operatorname{rank}\begin{pmatrix} A & 0 & B \\ C & 0 & 0 \\ 0 & I & 0 \end{pmatrix} = n+q_1+q_2 = n+2q$

定理 4-8. 设 $K_1 K_2 K_3$ 使得 (B.) 的特征值均有负实部. 干扰与输入均为斜坡.

即 $d(t) = \bar{d}t \cdot 1(t)$, $y_r(t) = \bar{y}_r t \cdot 1(t)$, 则 $\dot{x}(t)$. $\dot{q}_1(t)$. $\dot{q}_2(t)$ 均趋于常量. $y(t)$ 趋于 $y_r(t)$

证明：对 (B.) Laplace 变换.
$$\begin{pmatrix} X(s) \\ q_1(s) \\ q_2(s) \end{pmatrix} = \begin{pmatrix} sI-(A+BK_1) & -BK_2 & -BK_3 \\ -C & sI & 0 \\ 0 & -I & sI \end{pmatrix}^{-1} \begin{pmatrix} d(s) \\ -y_r(s) \\ 0 \end{pmatrix}$$

$d(s) = \bar{d} \cdot \frac{1}{s^2}$, $y_r(s) = \bar{y}_r \cdot \frac{1}{s^2}$

由终值定理，
$$\lim_{t\to\infty} \begin{pmatrix} \dot{x}(t) \\ \dot{q}_1(t) \\ \dot{q}_2(t) \end{pmatrix} = \lim_{s\to 0} s \cdot \begin{pmatrix} sX(s) \\ sq_1(s) \\ sq_2(s) \end{pmatrix} = \begin{bmatrix} -(A+BK_1) & -BK_2 & -BK_3 \\ -C & 0 & 0 \\ 0 & -I & 0 \end{bmatrix}^{-1} \begin{pmatrix} \bar{d} \\ -\bar{y}_r \\ 0 \end{pmatrix}$$

而 $\lim_{t\to\infty} \dot{q}_2(t) =$ 常量, 即 $\lim_{t\to\infty} e(t) = 0$.

4-11

$C_1B = [0 \ 0]$ ~~C_1B~~ $C_1AB = [28 \ -13]$ 故 $d_1 = 1$ $E_1 = [28 \ -13]$

$C_2B = [0 \ 0]$ $C_2AB = [2, 5]$ 故 $d_2 = 1$ $E_2 = [2 \ 5]$

$E = \begin{bmatrix} 28 & -13 \\ 2 & 5 \end{bmatrix}$ $|E| \neq 0$ 故可以用 $u = Kx + Hv$ 将闭环化为积分器解耦系统

$F_1 = C_1A^2 = [-5 \ 0 \ 9 \ -7]$ $F_2 = C_2A^2 = [-2 \ -4 \ 1 \ -2]$

$$F = \begin{bmatrix} -5 & 0 & 9 & -7 \\ -2 & -4 & 1 & -2 \end{bmatrix}$$

$$\therefore H = E^{-1} = \begin{bmatrix} \frac{5}{166} & \frac{13}{166} \\ \frac{-2}{166} & \frac{28}{166} \end{bmatrix} \quad K = -E^{-1}F = \begin{bmatrix} \frac{51}{166} & \frac{52}{166} & \frac{-58}{166} & \frac{61}{166} \\ \frac{46}{166} & \frac{112}{166} & \frac{-10}{166} & \frac{42}{166} \end{bmatrix}$$

[illegible]

$$A+BK=\begin{bmatrix} \frac{14}{83} & \frac{-2}{83} & \frac{15}{83} & \frac{20}{83} \\ \frac{28}{83} & \frac{-4}{83} & \frac{30}{83} & \frac{40}{83} \\ \frac{-60}{83} & \frac{56}{83} & \frac{78}{83} & \frac{104}{83} \\ \frac{38}{83} & \frac{-41}{83} & \frac{-66}{83} & \frac{-88}{83} \end{bmatrix} \qquad BH=\begin{bmatrix} \frac{3}{83} & \frac{-42}{83} \\ \frac{6}{83} & \frac{-1}{83} \\ \frac{-1}{83} & \frac{14}{83} \\ \frac{20}{83} & \frac{-31}{83} \end{bmatrix}$$

$$G_f(s)=\begin{bmatrix} \frac{1}{s^2} & \\ & \frac{1}{s^2} \end{bmatrix} \qquad \delta G_f(s)=4=d_1+d_2+p$$ ∴解耦与稳定不矛盾

4-13

a. $C_1B=[1\ \ 0]$ $\cancel{C_1AB=}$ $d_1=0$. $E_1=[1\ \ 0]$

$C_2B=[0\ \ 0]$ $C_2AB=[1\ \ 1]$ $d_2=1$ $E_2=[1\ \ 1]$

∴ $E=\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ $|E|\neq 0$. 可以动态解耦

$F_1=C_1A=[-1\ \ 0\ \ 0]$ $F_2=C_2A^2=[-4\ \ 4\ \ 5]$

∴ $F=\begin{bmatrix} -1 & 0 & 0 \\ -4 & 4 & 5 \end{bmatrix}$

∴ $H=E^{-1}=\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ $K=-E^{-1}F=\begin{bmatrix} 1 & 0 & 0 \\ 3 & -4 & -5 \end{bmatrix}$ $u=kx+Hv$

b. PBH不难判断，系统可控.

且 $\det\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}\neq 0$. 故可以静态解耦

设 $K=\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix}$ $A+BK=\begin{bmatrix} k_{11}-1 & k_{12} & k_{13} \\ k_{21} & k_{22}-2 & k_{23}-4 \\ 1-k_{21} & -k_{22} & 1-k_{23} \end{bmatrix}$

令 $k_{11}-1=-1$, $k_{22}-2=-2$, $1-k_{23}=-3$, $k_{12}=0$

~~其余=0~~, 则 $A+BK=\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{bmatrix}$

$k_{13}=0$, $k_{21}=0$

∴ $K=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$H=-[C(A+BK)^{-1}B]^{-1}M$, 取 $M=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, 则 $H=\begin{bmatrix} 1 & 0 \\ -2 & 6 \end{bmatrix}$

4-15

$C_1B=[1\ 0]$ $C_2B=[0\ 1]$ $E=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ 非奇异 故可以动态解耦.

$[\lambda I-A\ \ B]=\begin{pmatrix}\lambda&0&0&1&0\\0&\lambda&-1&0&0\\1&2&\lambda+3&0&1\end{pmatrix}$ 行满秩，故系统可控.

$\begin{bmatrix}A&B\\C&0\end{bmatrix}=\begin{pmatrix}0&0&0&1&0\\0&0&1&0&0\\-1&-2&-3&0&1\\1&1&0&0&0\\0&0&1&0&0\end{pmatrix}\begin{matrix}\leftarrow\\ \\ \\ \\ \leftarrow\end{matrix}$ 第2、5行相关. $\det\begin{bmatrix}A&B\\C&0\end{bmatrix}=0$，故不能静态解耦.

4-17 $C_1B=[1\ 0]$ $C_2B=[0\ 0]$ $C_2AB=[1\ 1]$ $\therefore d_1=0$ $d_2=1$

$E=\begin{bmatrix}1&0\\1&1\end{bmatrix}$ $\delta G_f(s)=3=d_1+d_2+p$ $\therefore$ 可以

$\therefore H=E^{-1}=\begin{bmatrix}1&0\\-1&1\end{bmatrix}$ $F_1=C_1(A+3I)=[2\ 0\ 0]$ $F_2=C_2(A^2+3A+2I)=[-1\ 0\ -2]$

$\therefore F=\begin{bmatrix}2&0&0\\-1&0&-2\end{bmatrix}$ $K=-E^{-1}F=\begin{bmatrix}-2&0&0\\3&0&2\end{bmatrix}$