Comparison between MUSIC and ESPRIT Direction of Arrival Estimation Algorithms for Wireless Communication Systems

Ousmane Abdoulaye Oumar, Ming Fei Siyau, Tariq P. Sattar

Faculty of Engineering, Science and the Built Environment, London South Bank University, London, UK
Email :{oumaro, siyaum, sattartp}@lsbu.ac.uk

Abstract— **this paper presents simulation of Angle of Arrival (AOA) estimation using the Multiple Signal Classification (MUSIC) and Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) algorithms. We study the localization techniques using the wireless communication systems and there are several algorithms that have the ability in calculate the Direction of Arrival (DOA) of the incident signals. We investigate and compare MUSIC and ESPRIT algorithms. The simulations indicate that the MUSIC algorithm is more accurate and stable compared to the ESPRIT algorithm.**

KEYWORDS— **DOA, AOA,MUSIC, ESPRIT, MATLAB.**

I. INTRODUCTION

The strong demand on usage of wireless communications has attracted larger number of developments. Nowdays the technology improvements have made the deployment of small, inexpensive, low-power, distributed devices, which are capable of local processing and wireless communication, a reality. Unlike traditional networks, sensor networks depend on dense deployment and co-ordination to carry out their tasks [1, 8].

Localization in Wireless Sensor Network (WSN) [9] has been developed in the last three decades, mostly being started as a military project and today have attracted significant research interest during the last few years. The Global Positioning System (GPS) (the result of heavy investments made by the U.S. Department of Defense in the 1970s) is an immediate solution however, there are some strong factors against the usage of GPS today [1, 7], such as indoor blind.

The most important method is flexible enough to perform localization using the most common radio measurements in wireless communication systems and various techniques for calculating the DOA, such as, time of arrival (TOA), time difference of arrival (TDOA), angle of arrival (AOA), and received signal strength (RSS).

We will study the methods of MUSIC (Multiple Classification Signal) and ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques).

MUSIC- based and ESPRIT-based algorithms, when used on uniform liner array with Omni-directional antenna elements, always have high performance [2, 3]. The accurate estimation of direction which is also known as DOA of the incident signals is very significant to produce beam form antenna. The DOA estimation techniques with array antennas are applied in wide areas of research fields and have received considerable attention in literature [11, 12].

MUSIC algorithm can estimate the signal's DOA accurately for any array geometry, but if the array's steering vector is incomplete e.g., when a circular array is mounted on a metallic cylindrical surface, for a signal with special DOA, some antenna elements cannot receive this signal [4]. Radiolocation can be implemented that are based on either signal strength or angle of arrival AOA measurements [5].

This paper is organized as follows. In Section II describes the DOA estimation methods. In Section III explains MUSIC and ESPRIT, in section IV explains in detail with numerical simulation examples and compared MUSIC and ESPRIT algorithms. Finally, section V gives conclusions.

II. DOA ESTIMATION METHODS

The estimation methods in WSN mainly include: RSS and time based methods (TOA, TDOA and AOA). In this paper, we mainly focus on AOA and to study MUSIC and ESPRIT

A. *Received signal strength (RSS)*

The distance between the transmitter and receiver can be estimated from the received signal strength associated with a propagation model in the environment. Three receivers are needed to determine the position in 2D (a GPS receiver needs at least three satellites to plot a rough, 2D position). By collecting multiple RSS measurements at different positions, the location of the target can be determined by the intersection of at least three circles area possible for the position of the moving object is deduced by trilateration, a process described in a later section.

This technique is easily applicable in the case of wireless local area network and cellular networks; the signal strength is available at the receivers and transmitter. The validity of the propagation model corresponding to the work environment plays an important role in the accuracy of the location. The superposition of multipath fading produced where it is very difficult to give a general model.

Depending on the configuration of the environment, multi-path leading to variations in signal level that can reach 15-25 dB over a distance of about a fraction of a wavelength. These random variations generate very large errors in estimating the distance. One possibility to improve results is to average the measurements over time or frequency and the measurement of power can also be combined with mapping techniques.

B. *Time of arrival (TOA)*

The TOA is one of the most popular techniques and most widely represented in the current localization systems. The signal's travel time between the target and observer can be expressed in a completely synchronized network as a result the distance can be calculated from the time of arrival as signals travel with a known velocity which in this case is the speed of light.

The distance between the target and observer, computed from the measured propagation time, provides a circle, centered at the observer's position, on which the target must lie. Placing the observers at three different locations, the target's position is given by the intersection of its corresponding circles [5].

This requires that the observers know the exact time at which the target will transmit, and that the observers have a very stable and accurate clock.

C. Time difference of arrival (TDOA)

The measurement of TDOA is to estimate the path difference of a signal observed at multiple receivers isochronous, regardless of the instant of transmission of this signal.

This can be achieved by finding the value of time shift that maximizes the cross correlation function between the signals received at a pair of references strictly synchronized. This generalized correlation method requires that the analog signal is received at first digitized and then transmitted to a central processing location. Unfortunately, the TDOA measurement cannot be obtained from the first local maximum observed in the correlation function, since the corresponding temporal shift does not coincide with the actual difference of arrival time. In addition, the delayed version of the correlation function resulting from true TOA does not necessarily coincide with the maximum of the inter correlation function. The result is sometimes a problem of ambiguity is difficult to remove. So far from the TDOA method of generalized cross-correlation is generally much more difficult than measuring direct TOA for dense multipath environments.

However, this measure TDOA can be easily obtained after differentiation from pre TOA Measures made at each receiver. The latter method, fairly intuitive, although it also suffers the problems of multipath and non-visibility, overcomes most of the ambiguity mentioned above, the objective being the systematic identification of the first trip observed on each of the observed responses. The measure of TOA is the most flexible and the most relevant solutions for the measurement of relative distances and TDOA measurements can be easily obtained from TOA measurements.

One way to access the TDOA is to measure the delay of signals arriving at each receiver and perform their difference. As usually in the case of systems using this technique, the receivers are not synchronized with the transmitter but only them, the TDOA measured and will include an offset which will be identical, however, because of the timing between the receivers. Another technique to estimate the TDOA is the correlation between signals received at different receivers.

D. Angle of arrival (AOA)

This technique is based on the use of angles of signals from the moving object at least two points of reception. The position of the transmitter is given by the intersection of lines passing through each receiver and the angle AOA calculated relative to an arbitrary reference. Often a margin of error of DOA estimation is introduced, leading to the intersection of the beams, which defines an area as possible to the position of the issuer. The size of each beam increases with distance from the transmitter leading to larger errors. In the case where the transmitter is on the line joining the two receivers, the position estimate is no longer possible. The presence of an extra receiver is needed and the major drawback of this technique is related to the need for antenna arrays that increase the size of the equipment used and involves additional costs.

III. Principle Analysis of MUSIC and ESPRIT Algorithms

The performance is analyzed on many parameters such as the number of users, number of snapshots, and number of array elements, user space distribution and signal-to-noise ratio (SNR). Parametric methods are based on simultaneous search parameters of interest and give as an estimate of point values. Complexity is higher but the results are more accurate.

A. Music algorithm

The MUSIC(Multiple Signal Classification)[6] method is one of the most used today in various treatment such as array processing, it uses the signals received by the M antenna sensors; these are the components signal X (t) from sources that are supposed to point and spatially coherent. The MUSIC method is based on the decomposition of specific elements of the covariance matrix is defined, nonnegative and Hermitian. The following equation shows this decomposition:

$$R = \sum_{m=1}^{M} \lambda_1 v_s V_m^H = V \Lambda V^H \quad (1)$$

Where the values of λ_m are real and positive, arranged in descending order, and the associated eigenvectors are orthonormal V_m .The absence of noise, the covariance matrix R is written:

$$R = ASA^H \quad (2)$$

Where the matrices A and S being the respective dimension (M, N) and (N, N), with N <M, the matrix R of dimension (M, M) has rank at most equal to N. In the preceding theorem we assume that A and S are full rank; hence R is of rank N. Moreover, R is a non-negative hermitian matrix defined, its eigenvalues are real and positive or zero. The assumption N <M implies that R M - N zero eigenvalues and N eigenvalues strictly positive.

Notes the subspace source subspace $\{V_s\}$ spanned by the eigenvectors associated with the N largest eigenvalues of R and the noise subspace $\{V_b\}$ subspace spanned by the eigenvectors associated with smallest eigenvalue σ^2. It thus comes as not fully correlated sources, it is the condition that S is full rank, the source vectors $a(\theta_n)$ are orthogonal to noise subspace, ie:

$$V_b^H a(\theta_n) = 0, n = 1,...N \quad (3)$$

For solving the nonlinear system gives uniquely the DOA sought, there must exist a number N of vectors of the form as an independent in N-dimensional space of M.

These observations form the basis of the MUSIC method. The algorithm is as follows: First, determining the minimum eigenvalue and the number of eigenvalues equal to this value. Then, as the noise subspace eigenvectors associated with the minimum eigenvalue then form the estimator:

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta) V_b V_b^H a(\theta)} \quad (4)$$

The MUSIC method is known for its superior performance and owes its popularity in part due to its use very general. Without limitation, it can be used for networks with arbitrary geometry known to estimate parameters from multiple sources both in terms of delay, angle, etc. In theory, if all assumptions are met, MUSIC allows parameter estimation asymptotically unbiased, the estimation error tends to zero if the number of observations tends to infinity.

B. Esprit algorithm

The ESPRIT method, introduced in [3] for the first time, is fast, efficient and robust parameter estimation can be used for determining the directions of incidence of multiple sources level of an antenna array. Due to its simplicity [10] and performance, ESPRIT has become a very popular method. The algorithm uses the same model of the signal that the MUSIC algorithm, but it has the advantage of significantly reducing the computing power and memory needed for storage. ESPRIT calculates the DOA and the time delay of'd' incoming signals by using their eigenvalues [3].Consider a network of 2M sensors fewer than two antennas deducted from each other by a translation of vector $\tilde{\Delta}$ known, translating the first sensor in the first antenna to the first sensor in the second antenna, this vector as module $|\Delta| = \Delta$. X_1 and X_2 are vectors of observation output of these two sub branches and let the vector X observing complete output of the antenna, and consequentially:

$$X = \left[X_1 \quad X_2\right]^T \tag{5}$$

The spectral matrices NxN and MxM sources of noise could be written as:

$$\Lambda = diag\left\{ e^{2\pi V_0 \Delta \frac{\sin\theta_1}{c}} + e^{2\pi V_0 \Delta \frac{\sin\theta_2}{c}}, \ldots, e^{2\pi V_0 \Delta \frac{\sin\theta_n}{c}} \right\} \tag{6}$$

Once the eigenvalues of λ_1 , $\lambda_2 \ldots \lambda_D$ are calculated, we can estimate AOA as:

$$\theta_i = \arcsin\left[-\lambda/_{2\pi\Delta}\right] \tag{7}$$

IV. SIMULATION RESULTS

In this section we investigate the performances of the MUSIC & ESPRIT algorithms using different parameters. A uniform linear array with M elements has been considered throughout our simulation experiments. It is assumed that the spacing between elements is 0.5λ which is required by these DOA algorithms to be effective. In case the distance is greater than the information contained in the phase shifts cannot deduct the angles accurately. The noise is considered as Gaussian white noise.

The simulation results of the MUSIC algorithm on two signals coming from two different AOA angles (25°, 125°) with 5 array elements (Fig.1) or 8 array elements and SNR=5dB (Fig.2) are given respectively. The results indicate clearly that if array size increases, the peaks for the spectrum become sharp and the resolution capacity of MUSIC increases.

The simulation results of the MUSIC algorithm on four signals coming from four different angles (60°, 80°, 90°, 100°) with 64 snapshots (Fig.3) or 100 snapshots (Fig.4) are given respectively. The results indicate clearly that if the number of snapshots increases (from 64 to 100), the resolution capacity of MUSIC increase and the four signals are clearly identifiable.

The simulation results of the MUSIC algorithm on four signals coming from four different AOA angles (45°, 90°, 110°, 160°), SNR=10dB, for snapshots 512, 16 array elements (Fig.5) and for snapshots 1024, 6 array elements (Fig.6) are given respectively. Also indicated clearly in the results that if the number of snapshots increases (from 512 to 1024), the resolution capacity of MUSIC increase and the four signals are clearly identifiable.

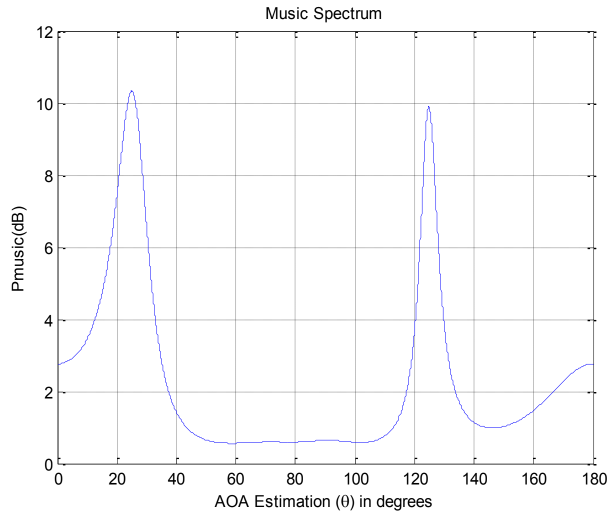

Fig.1. MUSIC spectrum for 64 snapshots, SNR=5dB and 5 array elements

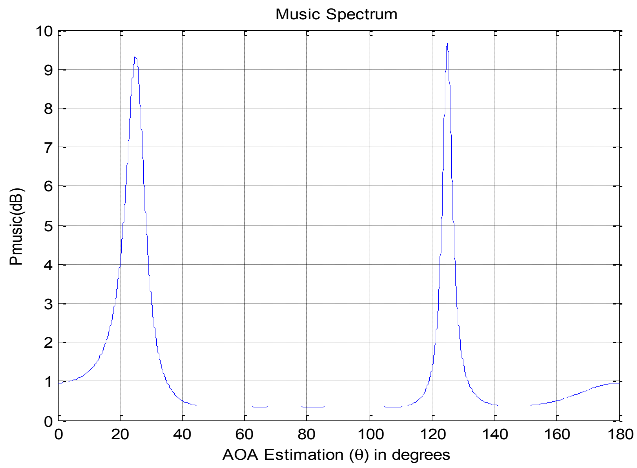

Fig.2. MUSIC spectrum for 64 snapshots, SNR=5dB and 8 array elements

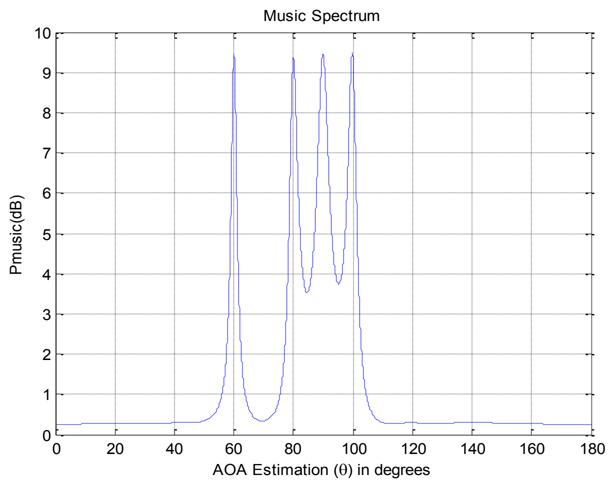

Fig.3. MUSIC spectrum for 64 snapshots, SNR=20dB and 10 array elements

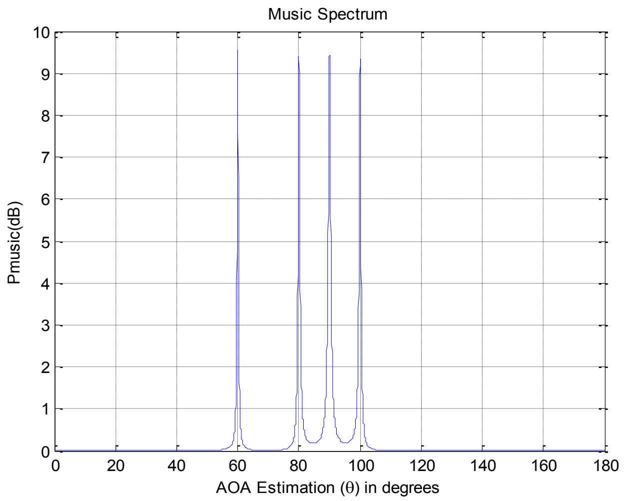

Fig.4. MUSIC spectrum for 100 snapshots, SNR=20dB, 10 array elements

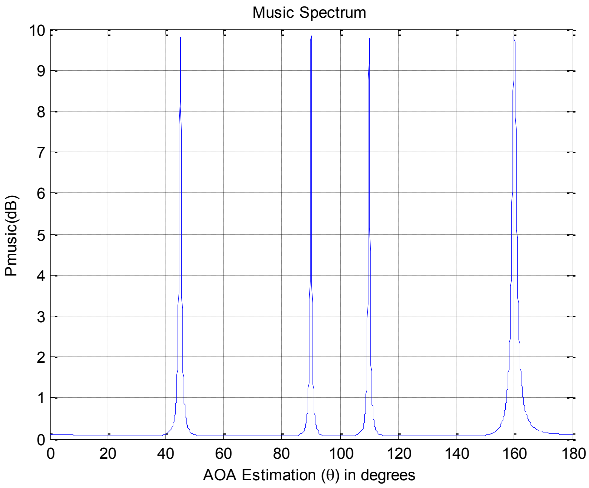

Fig.5. MUSIC spectrum for snapshots 512, SNR=10dB and 16 array elements

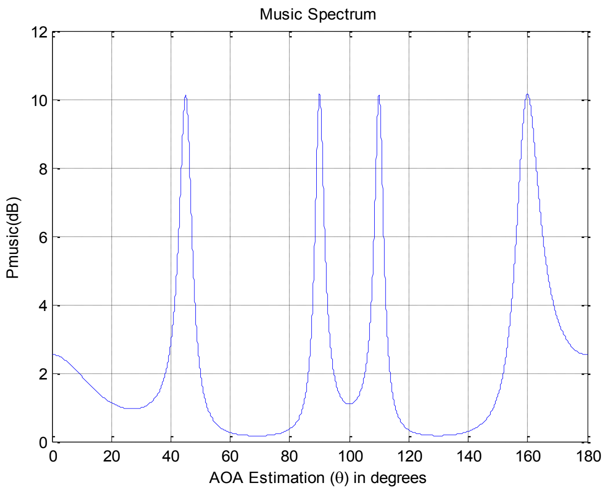

Fig.6. MUSIC spectrum for snapshots 1024, SNR=10dB and 6 array elements

The simulation results of the ESPRIT algorithm on four signals coming from four different AOA (60°,80°,90° &100°) with SNR=20dB (Fig.7) or SNR=5dB (Fig.8) and are given respectively. The results indicate clearly that the pictures expand and fade for low values of SNR and also that the accuracy of estimation deteriorates also when the noise dominates. The estimation error increases as the SNR decreases but remains less than 0.06° for the average levels of noise.

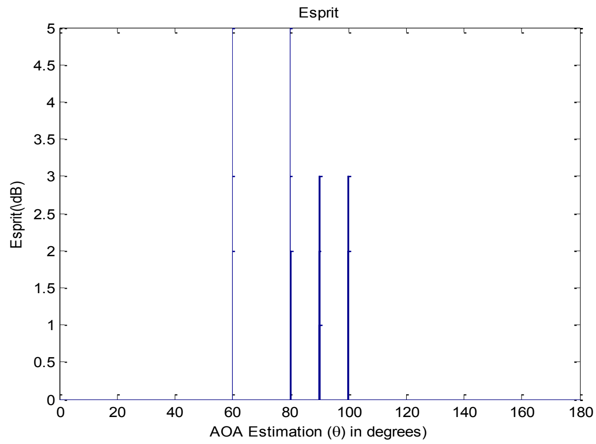

Fig.7. ESPRIT spectrum for 64 snapshots, SNR=20dB and 10 array elements

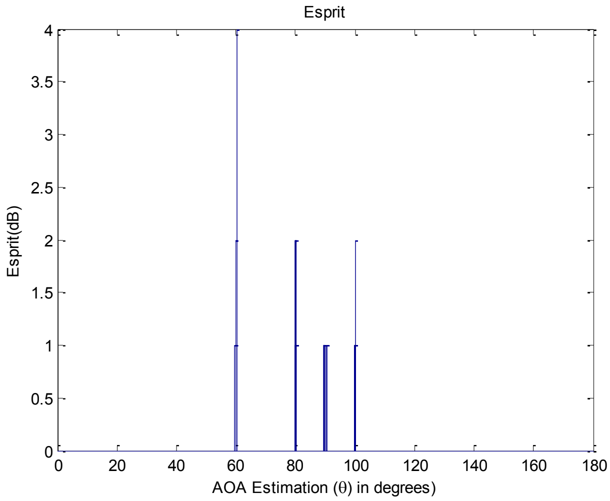

Fig.8. ESPRIT spectrum for 64 snapshots, SNR=5dB and 10 array elements

The AOA estimation results by MUSIC and ESPRIT for snapshots = 1024, M=10, SNR=20dB (Table 1) and for snapshot =64, SNR=5dB, M=10 (Table 2) are given respectively. In MUSIC & ESPRIT more antennas results in a higher spatial resolution, less antennas will results in the reduction of the spatial resolution and using less snapshots the signals are more correlated and increased snapshots results to MUSIC spectrum more accurate detection and better resolution than low snapshots. If the signals are in low level of correlation it will result in high peaks in the spectrum and in high level of correlation will result in small peaks in the spectrum. We see that the MUSIC is more accurate and high resolution than ESPRIT.

Table 1: DOA estimations (snapshots = 1024, M=10, SNR=20dB).

θ Input(degrees)	MUSIC(θ)	ESPRIT(θ)
60	60.0000	60.0027
80	80.0000	80.0029
90	90.0000	89.9906
100	100.0000	99.9815

Table 2: DOA estimations (snapshot =64, SNR=5dB, M=10).

θ Input(degrees)	MUSIC(θ)	ESPRIT(θ)
60	60.0002	59.0027
80	80.0000	80.0543
90	90.0000	90.0141
100	100.0000	100.0177

V. CONCLUSIONS

This paper compare MUSIC & ESPRIT algorithm based on the DOA. A presentation of two high resolution methods has been made in this paper. Attention has been paid to their performance in terms of SNR, snapshots and the number of antennas, in several cases of simulations. The results show that the MUSIC method is more accurate, stable and gives better resolution than the ESPRIT method.

REFERENCES

[1] A. Bharathidasan, V. Anand, S. Ponduru, "Sensor networks: An Overview", University of California, Davis, CA 95616.

[2] J. Hightower, Gaetano Borriello, "Location Systems for Ubiquitous Computing", IEEE Computer, August 2001

[3] R, Roy. and T. Kailath, ESPRIT "estimation of signal parameters via rotational invariance techniques," IEEE Trans. Acoust., Speech, Signal Process., Vol. 37, No. 7, Jul. 1989.

[4] P. Yang, F. Yang, and Z.-P. Nie, "DOA estimation with sub-array divided technique and interpolated esprit algorithm on a cylindrical conformal array antenna," Progress In Electromagnetics Research, Vol. 103, 201-216, 2010

[5] J. James, Jr. Caffery, and L. Stuber Gordon, "Overview of Radiolocation in CDMA Cellular Systems,IEEE Communications Magazine 36 (1998), pp.38 – 45

[6] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," Journal, IEEE Trans. Antennas Propagat., vol. 34, pp. 276–280, Mar. 1986.

[7] J. Hill, R. Szewczyk, A. Woo, S. Hollar, D. Culler, and K. Pister, "System Architecture Directions for Networked Sensors," Proceedings of the 9th International Conference on Architectural Support for Programming Languages and Operating Systems, November 2000.

[8] J. Yick, A. Bharathidasan, G. Pasternack, B. Mukherjee and D. Ghosal "Optimizing Placement of Beacons and Data Loggers in a Sensor Network – A Case Study", Department of Computer Science, University of California Davis and Department of Land, Air, and Water Resources, University of California Davis

[9] G. Mao, B. Fridan, B. Anderson, "Wireless sensor network localization techniques", Computer Networks, the International Journal of Computer and Telecommunications Networking archive Volume 51 Issue 10, July, 2007 Pages 2529-2553

[10] M. Haardt, J. A. Nossek, "Simultaneous Schur Decomposition of Several Nonsymmetric Matrices to Achieve Automatic Pairing in Multidimensional Harmonic Retrieval Problems", IEEE Transaction on signal processing, vol.46, No.1, Jan 1998.

[11] L.C. Godara, "Applications of antenna arrays to mobile communications," Proceedings of the IEEE, vol. 85, pp.1031-1060, July 1997 and Part II. Proc. IEEE, Vol. 85, No. 8, pp. 1195-1244, August 1997.

[12] B. Liao, S.C. Chan, "DOA Estimation of Coherent Signals for Uniform Linear Arrays with Mutual Coupling," Symposium on Circuits and Systems (ISCAS), 2011 IEEE International, May 2011, Rio de Janeiro, Brazil.