Effects of Target Displacement on Single-Snapshot DOA Estimation in Automotive Radar

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***Abstract*—This paper focuses on the effect of range velocity displacement of two targets on the accuracy of direction-of-arrival estimation in an automotive radar system. A simulative analysis is performed to evaluate the effect on the received signal phases for two targets with different positions, i.e. range and velocity displacement, within a range velocity cell. Experiments show an increasing error of the maximum likelihood estimation for two targets azimuth angle, as their spacing within the range velocity cell is increased. Finally the root cause and the impact of this behavior on real-world scenarios is analyzed.**

***Index Terms*—Maximum Likelihood, Direction-of-arrival estimation, Automotive Radar**

I. Introduction

In today's and future autonomous driving technology, radar sensors have become an important technology [1]. Frequency modulated continuous wave (FMCW) radar can accurately resolve targets in range and velocity. When multiple targets are within the resolution limit in range and velocity, spatial resolution becomes necessary. Therefore an antenna array is used to collect spatial information, from which the direction of arrival (DOA) and the number of the targets is estimated. All together, the three dimensions, range, velocity and angle, form the radar data cube [2]. The angular resolution depends on the aperture size of the antenna array, as well as the number of receive (RX) antennas. Since compact size and low cost are essential requirements for automotive radars, usually a maximum of four RX antennas is used in automotive radar [3]. High-resolution DOA estimation becomes a necessary method to break the physical resolution limits from those hardware constraints [4]–[6]. Maximum likelihood (ML) estimation or iterative approximations thereof have gradually become important algorithms used by automotive radar to predict DOAs [7]. A comparison of the performance of different algorithms in the case of single snapshot is performed in [8]. While the differences between correlated and uncorrelated target responses are mentioned in [8], we want to explicitly study in this work how the range velocity (RV) displacement of the two targets within the RV cell affects the observed phase differences at the antennas and hence might lead to errors for single-snapshot DOA estimation, as typically used in today's automotive radar sensors.

II. Problem Formulation

A. FMCW Radar Signal Processing

Fast-chirp FMCW radar can estimate range and velocity of targets utilizing Fourier-based signal processing. For a single measurement, a sequence of N_c linear frequency chirps is generated locally (LO), transmitted (Tx), and scattered back from the targets. The signal received (Rx) is then a linear superposition of all the individual target responses, which itself are delayed and attenuated copies of the Tx signal. Using the traditional approach for FMCW radar signal processing [7] consisting of downmixing the Rx with the LO signal, filtering the mixer output, digitizing the filter output using N_s samples per chirp, storing the data in an $N_s \times N_c$ radar-data matrix, and applying a 2D Fourier Transform along both dimensions, the complex-valued RV matrix of the measurement can be generated. Targets which are well-separated in range and/or velocity then appear as individual peaks in the magnitude of the RV matrix. Their presence as well as estimates of their range and velocity are typically formed by applying a simple peak detection, together with some post-processing to interpolate the peak position. If M antennas are used, M radar-data or RV matrices are available, which is referred to as the radar-data cube. Extracting the complex values corresponding to a target peak from those matrices, the DOA of the target can be estimated [8].

Targets whose displacement in range and speed is not sufficient appear as a single peak, i.e. they are not resolved in range or speed. Then the DOA estimation must be able to resolve the targets in the angular domain, i.e. estimate the number k of targets as well as their individual DOAs from the complex values extracted from that peak.

B. Data Model

We assume a uniform linear array (ULA) of M antennas with half-wavelength element spacing $d = \lambda/2$, receiving narrowband plane waves from $k < M$ sources (targets) in the far-field of the array. Additionally, we limit this work to azimuth angles only, referred to as θ. The signal vector $\boldsymbol{x}$ extracted from the RV matrices at a peak position can then be

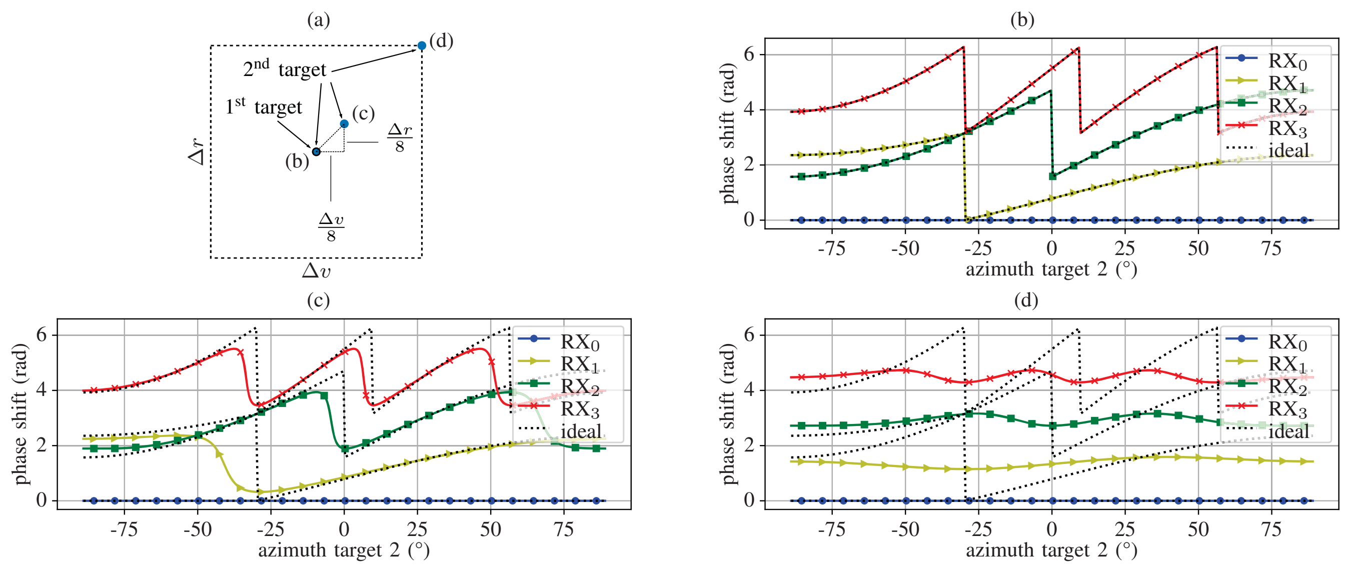

Fig. 1. Illustration of the target positioning within the RV bin (a) for the simulated scenarios (b)-(d). The resulting steering matrices, observed at the antennas $\mathrm{RX}_0 - \mathrm{RX}_3$, show an increasing deviation from the ideal model (dotted) where both targets are located in the center of the RV bin (b).

modeled by [3]

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}(\theta_1) & \boldsymbol{a}(\theta_2) & \dots & \boldsymbol{a}(\theta_k) \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_k \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_k \end{bmatrix} \tag{1}$$

or more compact by

$$\boldsymbol{x} = \boldsymbol{A} \cdot \boldsymbol{s} + \boldsymbol{n}, \tag{2}$$

where $\boldsymbol{A}$ denotes the steering matrix, $\boldsymbol{s}$ is the vector of complex signal amplitudes reflected by the k targets and $\boldsymbol{n}$ is spatially white noise with circular-complex Gaussian distribution. Assuming ideal omnidirectional point sensors, the matrix $\boldsymbol{A}$ consists of k column vectors $\boldsymbol{a}(\theta)$ which are related to the phase shifts at the M antennas for a certain DOA θ. Consequently, $\boldsymbol{A}$ can be further expressed as

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{j\pi\lambda\theta_1} & e^{j\pi\lambda\theta_2} & \dots & e^{j\pi\lambda\theta_k} \\ \vdots & \vdots & \dots & \vdots \\ e^{j(M-1)\pi\lambda\theta_1} & e^{j(M-1)\pi\lambda\theta_2} & \dots & e^{j(M-1)\pi\lambda\theta_k} \end{bmatrix}, \tag{3}$$

where λ is the wavelength at the center frequency of the FMCW chirp waveform. The spatial covariance matrix $\mathbf{R}_{xx}$ is a fundamental statistics in spatial signal processing containing all the information on the target DOAs. It is defined as

$$\boldsymbol{R}_{xx} = E\{\boldsymbol{x}\boldsymbol{x}^H\} \overset{L\to\infty}{\approx} \hat{\boldsymbol{R}}_{xx} = \frac{1}{L}\sum_l \boldsymbol{x}_l^H \boldsymbol{x}_l \tag{4}$$

where $E\{\cdot\}$ denotes the statistical expectation, $(\cdot)^H$ is the conjugate transpose and $\boldsymbol{x}_l$ are subsequent measurements (snapshots) with the target parameters unchanged. For a large number of snapshots L, $\boldsymbol{R}_{xx}$ can be well approximated using $\hat{\boldsymbol{R}}_{xx}$ [9]. In this paper our focus is on the effects in case of having a single snapshot $L = 1$ only, as this is the case for typical automotive radar applications where DOA estimation is performed individually for each measurement.

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C. Maximum Likelihood Estimation

Maximum Likelihood (ML) estimation is a well-known technique to estimate the DOAs θ_k of the targets. The estimate θ_k for the k-th target can be formulated by

$$\hat{\theta}_k = \arg\max_{\theta}\{Tr(\boldsymbol{P}_A(\theta)\boldsymbol{R}_{xx})\}, \tag{5}$$

where Tr denotes the matrix trace operator, $\boldsymbol{P}_A(\theta)$ is the projection matrix onto the column span of the steering matrix $\boldsymbol{A}$ [10] and we limit the attention to the case of two targets, $k \in [1, 2]$. Equation (5) is a nonlinear multidimensional maximization problem, thus ML estimation is computationally expensive [11].

D. The Specific Problem

We are interested in the very particular question: When using (5) to estimate the DOAs θ to the data model (1), how do the target responses from $\boldsymbol{s}$ influence the estimate? This question is of particular interest for practical automotive radar applications, where a single-snapshot DOA estimation is used. Since the targets indeed might occur as a single peak due to limited range and/or speed resolution, they of course would not have exactly the same range or speed. The difference in range and/or speed together with the scattering properties of the target give rise to a certain phase difference of the targets complex amplitude s_k. In the remainder of the paper, we study how this phase affects the ML estimate.

III. Simulative Analysis

Simulations are conducted to evaluate the influence of different target positioning on the phase shift in the received

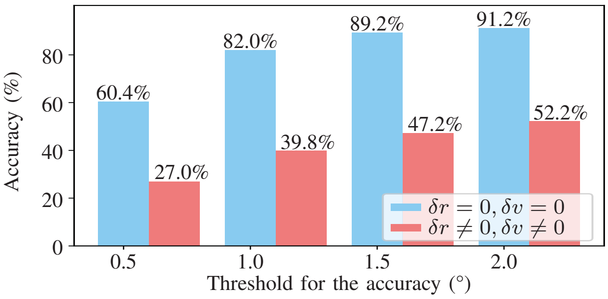

Fig. 2. Comparison of the total accuracy of the ML estimate without RV displacement of two targets (blue) and with RV displacement (red).

IF signal. Simulation data is generated by a simulation model for the full radar-data cube, based on a $M = 4$ element ULA with half-wavelength element spacing. The bandwidth of the simulated FMCW signal is $B = 800\,\mathrm{MHz}$, while $N_c = 256$ chirps and $N_s = 128$ samples per chirp are used with a sampling frequency of $f_s = 5\,\mathrm{MHz}$. The theoretical simulations are conducted without presence of noise and targets are simulated as point targets. The complex receive signal vector $\boldsymbol{x}$ is extracted from the radar-data cube at the target's RV bin. As only relative phase differences between different antenna elements are used to predict the DOA of targets, we normalize $\boldsymbol{x}$ to its first element x_1.

Three different scenarios are selected to illustrate the effect on the phase with increasing RV displacement. Fig. 1 (a) illustrates the geometry of the three test-cases (b)-(d). The first target is always fixed at the center of the RV cell (black border) with a fixed DOA of $\theta_1 = 30°$. The second target is simulated with three different offsets $\delta r, \delta v$ in range and velocity, relative to the first target, i.e. $\delta r = 0$ and $\delta v = 0$ (b), $\delta r = \Delta r/8$ and $\delta v = \Delta v/8$ (c) and $\delta r = \Delta r/2$ and $\delta v = \Delta v/2$ (d). The angle of the second target is then swept along the azimuth $\theta_2 \in [-90°, 90°]$ for each scenario. Fig. 1 (b)-(d) show the obtained phases $\phi \in [0, 2\pi]$ of the elements of $\boldsymbol{x}$ for the respective scenarios versus the azimuth of the second target. The black dotted lines depict the phases of the receive vector obtained from (3) assuming two targets θ_1 and θ_2 with $s_1 = s_2$. Notice the exact agreement of the simulated receive signal with the theoretical model for scenario (b) where both targets are located at the exact center of the range velocity cell. For scenario (c), the simulated phases already deviate from the model, depending on the exact DOAs. Increasing distance between both targets, within the same range velocity cell, results in increasing deviations as can be seen from scenario (d). As the second targets moves away from the center, its signal energy is spread across the neighboring bins and the total contribution of the second target to the received steering matrix is reduced. Clearly the fact that the targets do not have exactly the same position within the RV cell leads to a model error, whose effect on the DOA estimate will be studied in the next section.

IV. Statistical Evaluation

Previous work on single snapshot DOA estimation placed targets always in the center of RV cells, while accounting for potential range offsets by adding a random phase to the receive signals [10]. In this work, range and velocity of targets are randomly chosen from the RV map, while ensuring that both targets are always in the same RV cell. Angles of the targets are randomly chosen from $\theta_k \in [-90°, 90°]$, the SNR is set to $-10\,\mathrm{dB}$. The receive vector from the respective RV cell is taken and the covariance matrix is calculated according to (4). The maximum likelihood estimate is then obtained for the two targets azimuth angles θ_1 and θ_2. Fig. 2 shows a comparison of the calculated DOA estimation accuracy for different thresholds, which determine the maximum error for which an estimate is still considered correct. The blue bars are obtained when both targets always have the exact same range and velocity, thus $\delta r = 0$ and $\delta v = 0$. In total 500 different target combinations are used for the evaluation. The red bar shows the accuracy where both targets range and velocity are different from each other. The results confirm the initial hypotheses, as the ML estimation is more accurate for two targets with the same velocity and range. Additionally to the previous analysis, this also holds for targets which are not placed in the center of the bin. This can be explained, the signal energy of both targets is still spread over multiple bins, however as their range and velocity are exactly the same, the contribution to the total signal power of the selected bin is also equal and thus the ideal model with equivalent signal strength remains valid. If velocity and range are not the same, the accuracy will drop significantly, as the targets contribution to the total signal power are not equal.

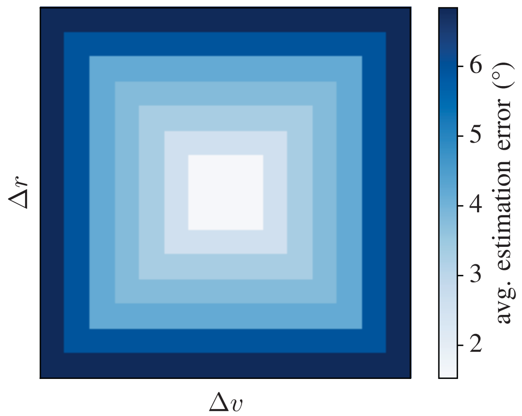

Fig. 3. The average ML estimation error increases with the RV displacement of two targets within the same RV cell.

A second experiment, evaluates the average estimation error for increasing target displacement δr and δv. Therefore, the range velocity cell is divided into seven smaller rectangles, compare Fig. 3. Then 500 data samples of two targets are randomly generated within each rectangle with random azimuth angles. Finally, the ML estimate for each data point is obtained and the average prediction error for each area is calculated.

The results confirm the expectation, where an increasing RV displacement of two targets within a RV cell will result in a greater average estimation error. The signal energy of two randomly placed targets is split over multiple RV bins, thus their contribution to the total signal energy will be unequal. As this is not accounted for in the ML model, it results in a decreased DOA estimation performance.

V. Conclusion

In this work, the effect of RV displacement on the receive signal of automotive FMCW radar is analyzed. Initial simulative analysis shows systematic deviations of the steering matrices for two targets, with an increasing target displacement. Real world targets will never have the exact same range and/or velocity, also radar cross section and thus reflection strength of the signal components will most likely be different. This results is an unequal contribution of both targets to the total signal energy in a RV cell and in turn severely decreases the accuracy of a ML estimation based on an ideal model with equal target strength.

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