A Survey of PHD Filter and CPHD Filter Implementations

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ABSTRACT

The probability hypothesis density (PHD) filter has attracted increasing interest since the author first introduced it in 2000. Potentially practical computational implementations of this filter have been devised, based on sequential Monte Carlo or on Gaussian mixture techniques. Research groups in at least a dozen different nations are investigating the PHD filter and its generalization, the CPHD filter, for use in various applications. Some of this work suggests that these filters may, under certain circumstances, outperform conventional multitarget filters such as MHT and JPDA. This paper summarizes these research efforts and their findings.

1 Introduction

The author introduced a multitarget detection and tracking approach called the *probability hypothesis density* (PHD) *filter* in 2000.[29,23] It has become the subject of investigation by researchers in over a dozen nations. The PHD filter is based on a *multitarget first-moment approximation.* Instead of the full multitarget probability distribution $f_{k|k}(X|Z^{(k)})$, it propagates a multitarget statistical first moment—the PHD $D_{k|k}(\mathbf{x}|Z^{(k)})$. The PHD filter has computational complexity $O(mn)$ where m is the current number of measurements and n is the current number of detected targets. Last year at this conference, and in response to a suggestion by Erdinc, Willett, and Bar-Shalom,[16] I introduced a generalization called the *cardinalized PHD* (CPHD) *filter*.[30,26] The CPHD filter propagates both the PHD and the entire probability distribution $p_{k|k}(n|Z^{(k)})$ on target number n (the *cardinality distribution*). It also admits more general false alarm models (so-called *i.i.d. cluster-process* models), more accurate estimates of target number, and more accurate state estimates. This additional capability comes at the price of greater computational complexity: $O(m^3 n)$. The CPHD filter is barely a year old, but like the PHD filter it also has inspired a number of independent research efforts.

The PHD and CPHD filters and methods for implementating them are described in detail in the new book *Statistical Multisource-Multitarget Information Fusion.*[27] Most implementations of the PHD and CPHD filters have been based on sequential Monte Carlo (SMC, a.k.a. particle-system) techniques. More recently, however, Vo and Ma and their colleagues have introduced Gaussian-mixture implementations that, while more restrictive than SMC approaches, tend to be much faster and generally easier to code.[58,56,51,52,32]

The primary aim of this paper is to provide a more up-to-date survey of current PHD/CPHD filter implementations than that provided in *Statistical Multisource-Multitarget Information Fusion.* On the basis of this reseach, a few preliminary implications can be drawn: (1) preliminary simulations suggest that versions of the PHD filter can outform MHT in cluttered environments, in the sense of having fewer false or dropped tracks;[32]

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(2) Gaussian-mixture implementations of the CPHD filter can outperform JPDA and IMM-JPDA in a cluttered environment, in the sense of achieving similar tracking performance but with much smaller computational load;[37] and (3) the PHD and CPHD filters have or are increasingly being implemented in real-time, real-data applications.

The paper is organized as follows. In Section 2 I begin with a very high-level overview of the PHD and CPHD filters, relegating all technical detail to the bibliography. Sections 3 and 4 present, respectively, surveys of implementations of the PHD filter and CPHD filter. Conclusions may be found in Section 5.

2 Review of Multitarget-Moment Filtering

This section reviews the PHD and CPHD filters at a high-level, non-mathematical level of detail. I begin, in Section 2.1, by revisiting the familiar concepts of second-order moment filtering (i.e., the Kalman filter) and first-order moment filtering (i.e., constant-gain Kalman filters) in the context of single-target tracking.

2.1 Single-Target Moment-Statistic Filters

I introduce the intuitive meaning of PHD and CPHD filters by revisiting the Kalman filter and its special case, the constant-gain Kalman filter. The Kalman filter presumes that signal-to-noise ratio (SNR) is high enough that all time-evolving posterior distributions $f_{k|k}(\mathbf{x}|Z^k)$ are not too complicated. That is, the $f_{k|k}(\mathbf{x}|Z^k)$ are essentially unimodal and not too skew for all $k \geq 0$. In this case one can *(lossily) compress* the posterior into *sufficient statistics* (first and second statistical moments) and propagate these statistics in place of the posterior itself. In the case of the Kalman filter the relevant statistics are the first-moment vector and second-moment matrix

$$\mathbf{x}_{k|k} = \int \mathbf{x}\ f_{k|k}(\mathbf{x}|Z^k)d\mathbf{x}, \qquad\qquad Q_{k|k} = \int \mathbf{x}\mathbf{x}^T\ f_{k|k}(\mathbf{x}|Z^k)d\mathbf{x} \tag{1}$$

where "^{T}" denotes matrix transpose. If SNR is high enough that higher-order moments can be neglected, $\mathbf{x}_{k|k}$ and $Q_{k|k}$ are approximate sufficient statistics in the sense that

$$f_{k|k}(\mathbf{x}|Z^k) \cong f_{k|k}(\mathbf{x}|\mathbf{x}_{k|k}, Q_{k|k}) = N_{P_{k|k}}(\mathbf{x} - \mathbf{x}_{k|k}) \tag{2}$$

where $N_{P_{k|k}}(\mathbf{x}-\mathbf{x}_{k|k})$ is a multidimensional Gaussian distribution with covariance matrix $P_{k|k} = Q_{k|k} - \mathbf{x}_{k|k}\mathbf{x}_{k|k}^T$. Given this assumption, we can propagate $\mathbf{x}_{k|k}$ and $P_{k|k}$ instead of the full distribution $f_{k|k}(\mathbf{x}|Z^k)$ using a Kalman filter. That is, we have a diagram of the form

$$\begin{array}{ccccccc} \cdots \rightarrow & f_{k|k}(\mathbf{x}) & \overset{\text{predictor}}{\rightarrow} & f_{k+1|k}(\mathbf{x}) & \overset{\text{corrector}}{\rightarrow} & f_{k+1|k+1}(\mathbf{x}) & \rightarrow \cdots \\ & \downarrow & & \downarrow & & \downarrow & \\ \cdots \rightarrow & \mathbf{x}_{k|k}, P_{k|k} & \overset{\substack{\text{KF} \\ \text{predictor}}}{\rightarrow} & \mathbf{x}_{k+1|k}, P_{k+1|k} & \overset{\substack{\text{KF} \\ \text{corrector}}}{\rightarrow} & \mathbf{x}_{k+1|k+1}, P_{k+1|k+1} & \rightarrow \cdots \end{array} \tag{3}$$

The top row portrays the prediction and correction steps of the single-target Bayes filter. The downward-pointing arrows indicate the compression of the posteriors into their corresponding first- and second-order moments. The bottom row portrays the prediction and correction steps for the Kalman filter.

If SNR is even higher, the second-order moment can be neglected as well. In this case the first moment is an approximate sufficient statistic

$$f_{k|k}(\mathbf{x}|Z^k) \cong f_{k|k}(\mathbf{x}|\hat{\mathbf{x}}_{k|k}) = N_P(\mathbf{x} - \mathbf{x}_{k|k}), \tag{4}$$

where now the covariance P is *constant*. We can propagate $\mathbf{x}_{k|k}$ alone using a constant-gain Kalman filter

(CGKF) such as the alpha-beta filter. In this case, the previous diagram reduces to

$$\begin{array}{ccccccc} \cdots\to & f_{k|k}(\mathbf{x}) & \stackrel{\text{predictor}}{\to} & f_{k+1|k}(\mathbf{x}) & \stackrel{\text{corrector}}{\to} & f_{k+1|k+1}(\mathbf{x}) & \to\cdots \\ & \downarrow & & \downarrow & & \downarrow & \\ \cdots\to & \mathbf{x}_{k|k} & \stackrel{\substack{\text{CGKF}\\ \text{predictor}}}{\to} & \mathbf{x}_{k+1|k} & \stackrel{\substack{\text{CGKF}\\ \text{corrector}}}{\to} & \mathbf{x}_{k+1|k+1} & \to\cdots \end{array} \tag{5}$$

2.2 The PHD Filter

Our goal is to extend this reasoning to the multitarget case. We assume that SNR is high enough that a first-order statistical moment $D_{k|k}$ of the *multitarget system* is an approximate sufficient statistic:

$$f_{k|k}(X|Z^{(k)}) \cong f_{k|k}(X|D_{k|k}). \tag{6}$$

We must then "fill in the question marks" in the following diagram,

$$\begin{array}{ccccccc} \cdots\to & f_{k|k}(X) & \stackrel{\text{predictor}}{\to} & f_{k+1|k}(X) & \stackrel{\text{corrector}}{\to} & f_{k+1|k+1}(X) & \to\cdots \\ & \downarrow & & \downarrow & & \downarrow & \\ \cdots\to & D_{k|k}(\mathbf{x}) & \stackrel{\substack{\text{PHD}\\ \text{predictor?}}}{\to} & D_{k+1|k}(\mathbf{x}) & \stackrel{\substack{\text{PHD}\\ \text{corrector?}}}{\to} & D_{k+1|k+1}(\mathbf{x}) & \to\cdots \end{array} \tag{7}$$

The top row portrays the multitarget Bayes filter. The downward-pointing arrows indicate the compression of the multitarget posteriors into their corresponding *first-order multitarget moments*. The bottom row portrays the time-prediction and data-update steps for the (as yet undefined) first-order multitarget-moment filter.

What is the first-order multitarget moment $D_{k|k}$ of $f_{k|k}$? It is not a vector, as in the single-target case. Rather, it is a *density function*

$$D_{k|k} \stackrel{\text{abbr.}}{=} D_{k|k}(\mathbf{x}) \stackrel{\text{abbr.}}{=} D_{k|k}(\mathbf{x}|Z^{(k)}) \tag{8}$$

defined on single-target states $\mathbf{x} \in \mathfrak{X}_0$. In point process theory $D_{k|k}(\mathbf{x})$ is called the *first-moment density* or *intensity density*. The author has, for historical and correct authorial-attribution reasons,[23] adopted the name *probability hypothesis density* (PHD). The PHD is not a probability density. In fact, it is uniquely characterized by the following property. Given any region S of single-target state space $\mathfrak{X}_0$, the integral $\int_S D_{k|k}(\mathbf{x})d\mathbf{x}$ is the expected number of targets in S. In particular, if $S = \mathfrak{X}_0$ is the entire state space then

$$N_{k|k} \triangleq \int D_{k|k}(\mathbf{x})d\mathbf{x} \tag{9}$$

is the total expected number of targets in the scene.

I have derived formulas for the predictor and corrector steps of the PHD filter indicated in Expression (7).[23,27] This filter is more kindred in spirit to joint probabilistic data association (JPDA) approaches than to multi-hypothesis correlation approaches. This is because it simultaneously associates all measurements with all tracks, rather than attempting to enumerate and rank a list of possible measurement-to-track associations.

2.3 The CPHD Filter

The potential limitations of the PHD filter should be evident to anyone who is aware of the superior performance of Kalman (second-order) compared to alpha-beta (first-order) filters in the single-target case. The possibility of second-order multitarget moment filters was investigated at an early stage.[24] Such a filter would propagate not only a PHD $D_{k|k}(\mathbf{x})$ but also a second-order multitarget moment—for example, a multitarget covariance density $C_{k|k}(\mathbf{x}, \mathbf{x}') \overset{\text{abbr.}}{=} C_{k|k}(\mathbf{x}, \mathbf{x}'|Z^{(k)})$. It is possible in principle to construct the predictor and corrector equations for such a filter. However, these are unlikely to be computationally tractable for problems involving more than a small number of targets.

The CPHD filter steers a middle ground between the information loss of first-order multitarget-moment approximation and the intractability of a full second-order approximation. In addition to propagating the PHD $D_{k|k}(\mathbf{x})$ of $f_{k|k}(X)$, it also propagates the cardinality distribution $p_{k|k}(n)$ of $f_{k|k}(X)$. That is, it propagates the *entire probability distribution on target number*. Propagation of $p_{k|k}(n)$ is much less computationally challenging than the propagation of a covariance density $C_{k|k}(\mathbf{x}, \mathbf{x}')$. As a result, the CPHD filter sidesteps most of the implementation difficulties of a full second-order multitarget-moment approximation.

As with the PHD filter, we must "fill in the question marks" in the following diagram:[26,27]

$$\begin{array}{ccccccc} \cdots \rightarrow & f_{k|k}(X) & \overset{\text{predictor}}{\rightarrow} & f_{k+1|k}(X) & \overset{\text{corrector}}{\rightarrow} & f_{k+1|k+1}(X) & \rightarrow \cdots \\ & \downarrow & & \downarrow & & \downarrow & \\ \cdots \rightarrow & p_{k|k}(n), D_{k|k}(\mathbf{x}) & \overset{\substack{\text{CPHD} \\ \text{predictor?}}}{\rightarrow} & p_{k+1|k}(n), D_{k+1|k}(\mathbf{x}) & \overset{\substack{\text{CPHD} \\ \text{corrector?}}}{\rightarrow} & p_{k+1|k+1}(n), D_{k+1|k+1}(\mathbf{x}) & \rightarrow \cdots \end{array} \tag{10}$$

3 Survey of PHD Filter Implementations

This section briefly summarizes applications to which PHD filter implementations have been applied. These include: tracking in terrain (Section 3.1); multiple moving-target tracking (Section 3.2); distributed tracking (Section 3.3); direction of arrival tracking (Section 3.4); active-acoustic tracking (Section 3.5); bistatic radio frequency tracking (Section 3.6); tracking in images (Section 3.7); sensor management (Section 3.8); joint tracking and classification with HRRR (Section 3.9); and group-target tracking (Section 3.10). "SMC-PHD" will refer to SMC implementations of the PHD filter, whereas "GM-PHD" will refer to Gaussian-mixture implementations.

3.1 Tracking Multiple Moving Targets in Terrain

Tracking targets in terrain is a nonlinear problem because of terrain constraints. Sidenbladh (nee Kjellström) describes an application of an SMC-PHD filter.[40] She compared the SMC-PHD filter with the multitarget SMC filter described in that section. Vehicles are of the same type and have three travel options: on-road, on-field, and in-forest, with a given *a priori* probability of traveling in each. The terrain map is used to construct a nonlinear motion model. The targets are observed by human observers who report estimates of position, speed, and direction. Such observations tend to have lower probabilities of detection but also low false alarm rates. In the simulations, three vehicles travel over roads, with one vehicle temporarily going off-road into a field. Since the PHD filter is a first-order approximation of the full multitarget Bayes filter, the latter would be expected to have better performance than the former. This proved to be true in the case of target number, which was more accurately estimated by the full filter. Position accuracy was comparable, though the PHD filter was more prone to temporary mistakes followed by rapid recovery.

3.2 Multiple Motion-Model Tracking

Punithakumar et al.[39] generalized the SMC-PHD filter to include multiple motion model techniques using jump-Markov switching. This is accomplished by sampling from the model transition probabilities as well as a conventional proposal density. Their jump-Markov model presumes two motion models: constant velocity and coordinated turn. The authors tested their multiple-model SMC-PHD filter against the two SMC-PHD filters which result from disabling one or the other of the two models. The two-dimensional simulation consisted of two targets moving along straight lines, occasionally punctuated by sharp turns. The targets were observed by a single range-bearing sensor with uniformly distributed false alarms. The multiple-model filter was significantly better at following the turns than either of the single-model filters.

The multiple motion model problem has also been addressed by Pasha et al.[36,37] They generalize their GM-PHD filter technique to include linear jump-Markov motion models. Their GM-PHD filter presumes three models: constant velocity, clockwise constant turn, and counterclockwise constant turn. They successfully tested the algorithm in two-dimensional simulations involving rapidly intertwining targets in a dense false alarm environment. In one simulation, five targets underwent rapid maneuvers with occasional tracking crossings and track osculations. A second simulation involved four targets, two of which have rapidly intertwining trajectories with frequent track crossings. The multiple-model GM–PHD filter successfully tracked the targets in both simulations. The GM-PHD filter was compared to the IMM-JPDA filter. It was found that the former "exhibits an unprecedented combination of good tracking performance and high computational efficiency."[37]

3.3 Distributed Tracking

Punithakumar et al.[38] devised and implemented a distributed SMC-PHD filter. This filter addresses the problem of communicating and fusing track information from a distributed network of sensor-carrying platforms, which detect and track a time-varying number of targets. They successfully demonstrated their approach in a simulation consisting of four computational nodes, sixteen bearings-only sensors, and multiple ground targets.

3.4 Direction of Arrival (DOA) Tracking of Dynamic Sources

Balakumar et al.[3] applied an SMC-PHD filter to the problem of tracking an unknown and time-varying number of narrowband, far-field signal sources, using a uniform linear array of passive sensors, in a highly non-stationary sensing environment. The authors note that conventional DOA techniques such as MUSIC fail in non-stationary environments, and that difficulties are only compounded when the number of sources can vary. The actual signal consists of outputs of the different sensors in the array, each of which is a superposition of the signals generated by the individual sources. Since this is a very different measurement phenomenology than that presumed by the PHD filter, the authors had to convert the actual measurement model to a detection-type model. They used discrete Fourier transform (DFT) techniques to determine coarse estimates of the DOAs, and these estimates were then employed as measurement-set inputs to the SMC-PHD filter. The filter was used to estimate the number of sources, as well as their DOAs and intensities.

The SMC-PHD filter was compared to another SMC filter[18] which was specifically designed to detect and track a varying number of narrowband signals in a nonstationary environment. It uses a random-jump Markov chain Monte Carlo (RJMCMC) technique to estimate target number and for resampling. It had previously been shown to outperform a conventional root-MUSIC approach. The two filters were compared in two simulations: one with fixed and one with varying target number. In the first simulation, two sources move near to each other without actually crossing. The authors found that the PHD filter "instantaneously" estimated the number of targets correctly without initialization; whereas the RJMCMC filter required 25 iterations to converge to the actual target number even with initialization. Furthermore, the SMC-PHD filter successfully separated the tracks as they moved closer together, whereas the RJMCMC filter estimated target number to be unity. The second simulation involved two appearing and disappearing targets. The first target entered the scene and disappeared after 3/4 of the scenario had elapsed. The second target appeared in the scene after 1/4 of the scenario had elapsed and remained thereafter. The two targets approach each other at approximately mid-scenario. The RJMCMC filter required 25 iterations to detect the first target, easily detected the second target when it appeared, but once again estimated target number as one when the targets neared each other. The SMC-PHD filter "instantly" estimated the correct number of targets even when they were closely spaced, and also correctly tracked the DOAs.

3.5 Active-Acoustic Tracking

Clark et al.[4,7] applied the GM-PHD filter to detection and tracking of underwater objects using forward-looking active sonar mounted on an autonomous underwater vehicle (AUV). Such objects can include marine animals and objects on the sea floor. Because a AUV is moving, even motionless objects on the bottom must be detected and tracked for purposes of collision avoidance and registration. The authors use image-segmentation techniques to extract high-reflectivity areas in sonar images. The centroids of the segmented regions are extracted as features and fed into the GM-PHD algorithm. The authors tested their algorithm on simulated sonar data and on real data collected by a 600 KHz sonar on an autonomous underwater vehicle (AUV). They also compared the performance of the GM-PHD and particle-PHD filters. The former significantly outperformed the latter. The authors attribute this to the better computational characteristics of the GM-PHD filter, as well as the greater ease of state estimation and track labeling associated with it.

Clark and Bell have extended their approach to 3-D active sonar.[8] They successfully tested their SMC-PHD filter on real data from an Echoscope forward-looking sonar with 64×64 beams. The data contained a single bottom-located target. They compared three state estimation schemes, including the EM algorithm and k-means. The k-means algorithm was not only faster than the EM algorithm, but also outperformed it.

3.6 Bistatic RF Tracking

Conventional radar tracking is based on radar systems in which the transmitter(s) and receiver(s) are located in a single apparatus. Bistatic radar tracking exploits the fact that radar transmissions reflect from airborne targets in all directions. Multiple receiving sensors, located at considerable distances from the actual transmitters, can parasitically collect RF reflections from airborne targets and use them to detect targets and infer their trajectories. Clutter objects called "ghost targets" result from spurious triangulations and must be eliminated as part of the tracking process. The more transmitter-receiver pairs that can be exploited and the more advantageous the transmitter-receiver geometries, the more effective the tracking and localization.

A closely related bistatic RF technique, passive coherent localization (PCL), employs continuous-wave sources (TV and FM radio stations) rather than pulsed-wave sources (radar).Tobias and Lanterman[45,44,46] have applied SMC-PHD filters to the PCL problem in reduced-complexity simulations. The EM algorithm was used to extract multitarget state estimates. Some success was achieved when both range and Doppler information were used.

3.7 Tracking in Images

Ikoma et al.[17] applied a SMC-PHD filter to the problem of tracking the trajectories of feature-points in time-varying optical images. A corner detector is applied to each frame in the video sequence to extract feature-points, which are then processed using the SMC-PHD filter. The authors successfully tested their algorithm on actual video images of a scene containing a walking person and a moving, radio-controlled toy car.

Wang et al.[60] applied SMC-PHD methods to tracking groups of humans in digital video. They use an adaptive background-subtraction method to extract objects of interest. The video background is adaptively estimated and then subtracted from the image pixels to produce foreground images. Morphological operations are performed on the foregrounds to remove noise. The resulting foreground consists of a number of "blobs" associated with the targets. The centroids of these blobs are fed to the SMC-PHD filter as its data. The authors tested their algorithms on real video images consisting of pedestrians entering, moving within, and exiting from a fixed camera field of view. They observe that their method can successfully track human groups in video.

3.8 Sensor Management

I have proposed a systematic approach to multisensor, multitarget sensor management[25] based on the PHD filter. The approach includes the ability to preferentially bias sensor collections towards targets of tactical interest, based on their relative rank order of priority.[25] El-Fallah et al. have successfully demonstrated SMC-PHD implementations of the approach in reduced-complexity simulations.[12,13,14,11] These simulations have shown that PHD-based sensor managers have the ability to distribute their looks so as to collect more information on poorly-resolved targets, while not losing too much information on better-resolved targets. These simulations have been oriented primarily towards optimal surveillance of detected targets. Further work will concentrate on extending the approach to optimal search for currently undetected targets.

3.9 Joint Tracking and Classification with HRRR

Zajic et al.[62] report an algorithm in which an SMC-PHD filter is integrated with a robust classifier algorithm which identifies airborne targets from high range-resolution radar (HRRR) signatures. The classifier algorithm utilizes wavelet features ϕ extracted from the HRRR signatures to produce a feature likelihood function $L_\phi(u)$ where u is target type. The classifier was trained on real HRRR data sets drawn from seven possible airborne target types. Assuming that target identities and kinematics are independent, the total likelihood function has the form $L_{\mathbf{z},\phi}(c, \mathbf{x}) = L_\phi(c) \cdot L_\mathbf{z}(\mathbf{x})$. This was then incorporated into an SMC-PHD filter algorithm. The joint identification and tracking algorithm was tested on real HRRR signatures and simulated two-dimensional kinematics. The dynamics measurements included moderately dense false alarms (120 false alarms per frame). The classifier was not turned on until after the PHD filter had achieved good track estimates. The algorithm successfully detected, tracked, and classified the two targets.

3.10 Group-Target Detection and Tracking

Ahlberg et al. have employed PHD filters for group-target tracking in an ambitious situation assessment simulator system called IFD03[2] A force aggregation algorithm based on Dempster-Shafer methods is used to detect and classify force structure. Separate PHD filters are used to track group targets at each force structure level. The PHD filters can take account of *a priori* information derived from detailed terrain simulators.

It should also be mentioned here that I have proposed[21,22] a generalization of the PHD filter, the "group-PHD filter," for detecting and tracking group objects such as squads, platoons, brigades, etc. To my knowledge, there are no published implementations of this filter.

4 Survey of CPHD Filter Implementations

Since the CPHD filter was first introduced in 2006,[30,26] only a few implementations have appeared.

4.1 Basic Simulations

Vo, Vo, and Cantoni[59] announced the Gaussian-mixture implementation of the CPHD filter. They also described two versions of the GM-CPHD filter: one using an extended Kalman filter (EKF) and another using the unscented Kalman filter (UKF), which is capable of operating under nonlinear conditions. They tested and compared these two versions in two-dimensional simulations. In one simulation, five targets appeared and disappeared while observed by a linear-Gaussian sensor in a dense Poisson false alarm environment. The EKF version of the GM-CPHD filter correctly detected all target births and deaths and successfully tracked the targets during the times they were present in the scene. It also successfully negotiated a track crossing which occurred at mid-scenario. A second scenario also involved five targets with target appearances and disappearances. This time, however, the sensor was nonlinear (range-bearing) and nonlinear target dynamics occurred. Both the EKF and UKF versions of the GM-CPHD filter were tested on the simulated data. The two versions both successfully negotiated the simulation with similar performance.

Vo et al. also described detailed performance comparisons of the GM-PHD and GM-CPHD filters.[56] In the first scenario, up to ten targets could appear randomly, with track crossings. The GM-PHD and GM-CPHD filters were both successful at identifying target births and deaths, at tracking the targets, and at negotiating track crossings. As expected, for any individual sample path (measurement-sequence), the GM-CPHD filter's estimates of instantaneous target number were far more accurate and stable (small variance) than those of the GM-PHD filter.

However, the authors also noted unexpected differences between the sample-path and the Monte Carlo behaviors of the two filters. The GM-PHD filter's Monte Carlo instantaneous estimates of target number were essentially as accurate as those for the GM-CPHD filter. (For any particular sample path, of course, the PHD's instantaneous target number estimates would be much worse.) Moreover, the GM-PHD filter's Monte Carlo response to target appearance and disappearance was "almost instantaneous"; whereas the GM-CPHD filter's Monte Carlo response was sluggish. Vo et al. speculated that this behavior was due to the fact that the PHD filter has a weak memory and thus is easily influenced by new measurements. I suspect, however, that this only partially explains the results. Because the CPHD filter has a better memory—i.e., its effectiveness as a filter is better—its behavior is more heavily influenced by its underlying multitarget motion model for birth and death. If actual target birth and death in a scenario deviates from this model, then the CPHD filter will tend to respond sluggishly in an averaged, Monte Carlo sense. The PHD filter, which is less influenced by the motion model because of its more limited memory, will have better transient response in a Monte Carlo (but not a sample-path) sense. It is possible that the GM-CPHD filter would exhibit better Monte Carlo transient response if it were capable of adaptively choosing its internal birth-death models using jump-Markov techniques.

4.2 Comparison with JPDA

Vo et al. have also conducted apples-with-apples comparisons of the Gaussian mixture CPHD filter with joint probabilistic data association (JPDA) trackers. Since JPDA presumes that target number has already been estimated using some procedure, the simulations were conducted under this assumption. It was found that the GM-CPHD filter performed as well as JPDA, but with much smaller computational load.[56]

4.3 GMTI Ground-Target Detection and Tracking

Ulmke et al. have applied the Gaussian-mixture CPHD filter to the problem of detection and tracking of road-constrained ground targets using ground moving target indicator (GMTI) radar.[47] This required extending it to incorporate digital road maps as *a priori* information, as well as state-varying GMTI probabilities of detection (associated with, for example, low Doppler and terrain obscuration). They also modeled target dynamics in terms of "road coordinates." Various simulations were carried out assuming a single, static GMTI radar. They verified that CPHD filter performance was greatly improved if *a priori* terrain information (roads) was incorporated into the dynamics model. They also demonstrated that the CPHD filter was capable of tracking target groups, and accurately estimating the number of vehicles in them, even when vehicles were too closely spaced to be resolved and tracked individually. Ulmke et al. also showed that, when the number of targets cannot exceed one, the CPHD and MHT filters are mathematically identical.

5 Conclusions

The PHD and CPHD filters have inspired research efforts in at least a dozen nations. In this paper I have summarized those of which I am aware. Some of this work suggests that these filters may, under certain circumstances, outperform conventional multitarget filters such as MHT and JPDA. Real-data implementations are beginning to emerge, in areas as diverse as underwater active acoustics and air-to-ground GMTI detection and tracking. Future research will determine the range of applications for which these filters are best suited.

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